ABSTRACT
The Mathematica environment allows the user to describe visualized objects in the form of mathematical formulas and expressions. Exporting geometry as 3d objects allows further exploration in other software applications. These two features give Mathematica a vast potential for creative applications in fields of design and architecture.

This paper illustrates a design approach that uses Mathematica as a design tool. The paper explains some of the key techniques employed in the architectural design research project, Hybrid Species. Hybrid Species works on generating an interweaving spatial organization as an alternative spatial layout for corporate building type. The project’s research focuses on developing of morphing and manipulating techniques of Triply Periodic Minimal Level Surfaces (TMPLS).

INTRODUCTION

Hybrid Species is a design research project that rethinks the office building type by reflecting on the boundary condition between two distinct organizations coexisting within an office block of London’s financial district. Hybrid Species employs a mathematical approach for developing complex spatial relationships as means for re-interpreting the corporate building type.

The term hybrid specie describes a topological enactment achieved through homotopy. Homotopy is the measure of a function’s topology in parametric morphing operations where it is said that two mathematical functions are homotopic if one can be continuously deformed into the other. Mathematical objects such as surfaces can be divided into species each sharing a distinctive topology or equation. Hybrid Species utilizes equations of Triply Periodic Minimal Level Surfaces (TPMLS) and a software application Mathematica, to calculate and visualize the topological transformations between species creating new surface topologies called hybrids.

The graphic output produced by Mathematica includes 2d and 3d graphic plots which can be exported in .dxf format making it accessible in other software applications such as Max, Rhino, AutoCAD, etc. The built-in functions for visualization that were mostly used in this project include: Plot, PolarPlot, ParametricPlot, DensityPlot, CounterPlot, ImplicitPlot, Plot3d, ParametricPlot3d, CounterPlot3d and TablePlot for animations.
TPMLS – GENERAL CHARACTERISTICS

Minimal surfaces are defined within the language of differential geometry as surfaces of zero mean curvature. This means they are equally convex and concave at all points and their form is therefore saddle-like, or hyperbolic. They are called minimal because given a fixed boundary curve the area of a “minimal surface” is minimal with respect to other surfaces with the same boundary. A soap film minimizes its area under surface tension, so dipping a wire frame into soapy water produces a minimal surface geometry.

Several triply periodic level surfaces – TPLS defined using trigonometric functions closely approximate certain triply periodic minimal surfaces – TPMS and so they form the triply periodic minimal level surfaces – TPMLS.

Main Properties

TPMLS main properties – surfaces with complex geometry and periodic character generating two continuously interweaving volumes.

Basic Surfaces

The gyroid, illustrated above, is an infinitely connected periodic minimal level surface containing no straight lines and it is the only known embedded triply periodic minimal surface with triple junctions. In addition, unlike the five triply periodic minimal surfaces shown here, the gyroid does not have any reflectional symmetries.³

Diagrams of a Gyroid showing two continuously interweaving volumes
DESIGN APPROACH

The investigation into TPMLS stems out of the project’s position on the effect of corporate building type on planning and zoning constraints of urban architecture. It asserts that the repetitive nature of the horizontal slab division of buildings into autonomous slices of real-estate has dominated the greater part of the 20th century corporate architecture. The project defines the corporate building type as a slab on column construction system whereby vertical and horizontal planes subdivide the building for use by distinct organizations. The design problem being posed here asks to conceive two formally-different spaces of real-estate that can support a side by side coexistence of two essentially different organizations no longer bound by the conventional rules described in the corporate building type.

The characteristics of TPMLS described earlier, provide a unique way of thinking about spatial interviewing within a single surface. To better understand the spatial qualities of TPMLS requires more than one method of representation. One of the difficulties in describing the TPMLS without resorting to mathematics, is that it is almost impossible to represent the surfaces solely with architectural conventions such as plans and sections. Often time animations are required to understand the spatial qualities of these surfaces as well as concepts such as morphing and other mathematical manipulations.

The concept behind Hybrid Species is described in an animation that illustrates salient formal strategies used in the project. The project is formulated using different levels of morphing applied to the TPMLS. The levels of morphing are outlined below.

**Level 0.0:**
[Assigning Scale of Pi]
Part of the plane surface is assigned scale of $2\pi \times 2\pi$ where $3.14m = \pi$. A

**Level 1.0 Morphing:**
[Plane Surface to TPMLS]
Interweaving of two spaces = Bicontinuous Space
Level 2.0 Morphing:
[TPMLS Offset]
Introduction of an interstitial space = Tri continuous Space

Level 3.0 Morphing:
[Offset Articulation]
Differentiation in scale / topology.

TPMLS – CONTROLLED IN MATHEMATICA

Basic formulas and modules
The three variables in the TPMLS formulas: x, y, z are independent in regards with each other and they are presented there in different combination of their trigonometric functions. When a TPMLS formula equals zero or any other number, the repetitive character of the functions appears due to the periodic character of the trigonometric functions.

Methods of control
Regularity and symmetry are inherent properties of the TPMLS. For this reason, the project considers introducing more spatial differentiation to the surfaces that can create spatial richness and begin to operate closer to architectural line of thought.

In order to break away from the ‘pure’ state of the TPMLS the research focuses on developing means of controlling the surfaces mathematically.

The methods of control used in this study can be organized in five different techniques in the order of progression.

1. Adding & Subtracting.
2. Multiplying & Dividing.
3. Mixing Terms.
4. Combinations.
5. Hybridising.

The most significant change in the level of differentiation occurs in the Hybridising technique. The four other techniques, on the other hand, preserve the ‘pure’ surface qualities from the equation. For example, in Mixing Terms surfaces are mixed with each other to produce new surface families. This is achieved by using TPMLS terms in the equations, creating surfaces that maintain properties of both terms. For a given pair of such terms, a 2–parameter family can be generated by parameterizing the equation with variables s and t as,

\[ 0 = s \cdot \text{term1} + (1 - s) \cdot \text{term2} + t; \]

so that s gives the relative weights of the two terms and t gives an offset.

Whereas Hybridising, generates hybrid surfaces that poses new characteristics not found in the parental equations of the TPMLS. Hybridization is achieved by insertion of functions inside variables of TPMLS. This operation, which does not exist in any of the other methods, is the main difference in the way of approaching topological morphing of TPMLS. More explicitly, this control has been gained by the realization of what type of function should be inserted in which variable of the TPMLS formula.

The new variables can be named as the basic mathematical functions, namely the Square Root, Exponential, Natural Logarithm, Factorial, and Absolute Value. These functions are inserted in various places in the parental TPMLS functions, thereby creating a range of geometries that individually have unique properties. By exploring how these new variables manipulate the surfaces when inserted in specific parts of the functions, in that respect, this research investigates the possible architectural properties of TPMLS by producing a number of spatial studies taking into account the use of these new variables in different parts of the TPMLS formulas.
1. Adding.
\[ h = f + g \]
\[ h(x, y, z) = f(x, y, z) + g(x, y, z) \]

\[ h = s \cdot f + (1 - s) \cdot g \]
\[ h(x, y, z) = s \cdot f(x, y, z) + (1 - s) \cdot g(x, y, z) \]

2. Multiplying.
\[ h = f \cdot g \]
\[ h(x, y, z) = f(x, y, z) \cdot g(x, y, z) \]

4. Combinations.
\[ h = u^t + v^g \]
\[ h(x, y, z) = u(x, y, z) + v(x, y, z) \cdot g(x, y, z) \]
consideration the volume/surface area, volume/occupiable area, cross-sections, and slope analysis that were applied to certain Hybrid Surface Species.

In the preceding diagrams, the Gyroid (G) and Diamond (D) surfaces has been used as examples in explaining methods described above.

**TPMLS – SPACE GENERATION**

The process of generating architectural space used in Hybrid Species can be divided into three separate stages: distribution, differentiation and enclosure.

**Distribution**

Mathematica environment utilizes limits or bounding boxes when plotting functions. It is necessary to input limits for each direction – xmin, xmax, ymin, ymax, zmin, zmax. Using this as a principle for constraining the output, a grid defining overall boundaries of the site is extracted from a digital model in 3ds Max. The grid is extracted from the edges of the existing buildings. Once the general volume is complete it is subdivided into modules of Pi so that it can be replicated in Mathematica language.
This volume is then subdivided according to use, function and general program – circulation core, entrance lobbies, work areas, etc. Hybrid species are then generated in mathematica to best fit the requirements mentioned above. Each surface is produced in relationship to its adjacent surfaces and is part of a homotopic function. The surfaces are exported as .dxf files to 3ds Max, where they are placed in their respective position. Each side of the surface is given a color distinguishing Space A from Space B.

To achieve difference in height of volumes within the main building block the volume is divided in sections as follows: 2Pi, 3Pi, 2Pi, 4Pi. One overall morphing operation is applied at the beginning of the manipulation process that holds all the surfaces in one sequence. The secondary treatment is more distinctive for selected regions.

**Differentiation**

The differentiation stage involves generating differentiation in the spatial properties of Space A and Space B. The project speculates on the side

3. GymBox Offset.02
s = 0.55
GymOffset02=ContourPlot3D[
s (0.67) (Cos[Sqrt[9Pi-x]])
Sin[1/(4Pi-y)/2])
Cos[Log[1/(4Pi-y)/2]] Sin[(z+Pi/2)/1.5]
Cos[(z+Pi/2)/1.5] Sin[Sqrt[9Pi-x])])
(0.33) (Cos[(z+Pi/2)/1.5]
Sin[Log[(x+2Pi)^2/2]] Sin[(y+4Pi)^1/2]
Cos[(y+4Pi)^1/2] Sin[(z+Pi/2)/1.5]+ 0.2) - (1 - s) (Cos[x/2] Cos[y/2]+ Cos[x/2] Cos[z/2-Pi/4]+ Cos[z/2-Pi/4])
Cos[x/2-Pi/2] Cos[x/2]-0.5 Cos[x/2] Cos[y/2] Cos[x/2-Pi/4]+0.2),
{x, Pi, 9Pi}, {y, 0, 4Pi}, {z, Pi, 4Pi}]

4. Wofram Offset.02
s = 0.85
WolfOffset02=ContourPlot3D[
s (0.75) (0.55) (0.67) (Cos[Sqrt[9Pi-x]])
Sin[1/(4Pi-y)/2])
Cos[Log[1/(4Pi-y)/2]] Sin[(z+Pi/2)/1.5]
Cos[(z+Pi/2)/1.5] Sin[Sqrt[9Pi-x])])
(0.33) (Cos[(z+Pi/2)/1.5] Sin[Log[(x+2Pi)^2/2]])
Cos[Log[(x+2Pi)^2/2]] Sin[(y+4Pi)^1/2]
Cos[(y+4Pi)^1/2] Sin[(z+Pi/2)/1.5]+ 0.2) - (1 - s) (Cos[x/2] Cos[y/2]+ Cos[x/2] Cos[z/2-Pi/4]+ Cos[z/2-Pi/4])
Cos[z/2-Pi/4] Cos[x/2]-0.5)
(0.45) Cos[x/2] Cos[y/2]+ Cos[x/2] Cos[Z/2-Pi/4]+ Cos[x/2-Pi/4] Cos[x/2]-0.5 Cos[x/2] Cos[2Pi/4]+0.2-
(0.25) Cos[x/2] Cos[y/2]+ Cos[x/2] Cos[z/2-Pi/4]+ Cos[z/2-Pi/4] Cos[x/2]-0.5)
(0.5) Cos[x/2] Cos[y/2]+ Cos[x/2] Cos[z/2-Pi/4]+ Cos[z/2-Pi/4] Cos[x/2]-0.5)
(0.5) Cos[x/2] Cos[y/2]+ Cos[x/2] Cos[z/2-Pi/4]+ Cos[z/2-Pi/4] Cos[x/2]-0.5)
(0.5) Cos[x/2] Cos[y/2]+ Cos[x/2] Cos[z/2-Pi/4]+ Cos[z/2-Pi/4] Cos[x/2]-0.5)
(0.5) Cos[x/2] Cos[y/2]+ Cos[x/2] Cos[z/2-Pi/4]+ Cos[z/2-Pi/4] Cos[x/2]-0.5)
(0.5) Cos[x/2] Cos[y/2]+ Cos[x/2] Cos[z/2-Pi/4]+ Cos[z/2-Pi/4] Cos[x/2]-0.5)
(0.5) Cos[x/2] Cos[y/2]+ Cos[x/2] Cos[z/2-Pi/4]+ Cos[z/2-Pi/4] Cos[x/2]-0.5)
by side coexistence of two organizations with opposing briefs and spatial needs. For example, Space A (yellow) is for an organization that requires primarily large, open spaces. On the contrary, Space B (blue) requires small, cellular distribution of space.

Differentiation is achieved in Mathematica by applying additional manipulation to the basic surfaces constructed in the distribution stage. First, the basic surfaces are offset on a distance creating an interstitial space separating Space A and Space B. Second, additional morphing of the offsets is performed to introduce specific spatial characteristics independent to each color.

**Enclosure**

One characteristic that describes the TPMLS is that they are infinitely continuous. When plotted in Mathematica the surfaces are ‘cut’ by the bounding box of the plot, and never appear to have self enclosure. The need for enclosure such as the building facade requires an additional treatment of a surface edge. In order to provide the building elevation with enclosure a vertical slice of 2Pi, starting from the street building edge, is cut from the differentiated surfaces. The slice is then morphed with a plane to create enclosure. This technique creates a new slice that when placed back to its position it maintains the spatial continuity of the building.

**CONCLUSION**

The design research project, Hybrid Species is part of an ongoing exploration into Triply Periodic Minimal Level Surfaces that uses Mathematica as a design tool. The Mathematica software is used to calculate and visualize the topological transformations between species creating new surface topologies called hybrids. This approach to architectural design offers an interesting alternative to the planning and zoning of corporate buildings. The techniques illustrated in this paper represent only a few key methods of mathematical control used for this project. The development and research of other methods of controlling mathematical topologies such as TPMLS has a great potential for creative applications in fields of design and architecture.
Morphing wire model

Curvature analysis of specie01 Prototype of specie01, smooth surface

Tetrahedron construction of specie01 Prototype of specie01, tessellated surface

1:100 STL model in context 1:200 SLA model in context
YME presents 'Hybrid Species' at the Architectural Association in London
END NOTES


3. Karcher, Hermann, Polthiery Konrad Construction of Triply Periodic Minimal Surfaces, University at Bonn Mathematics Institute, Bonn Germany, (4.01.1996)